

# Interacting Vector-like Dark Energy, the First and Second Cosmological Coincidence Problems

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## ABSTRACT

One of the puzzles of the dark energy problem is the (first) cosmological coincidence problem, namely, *why does our universe begin the accelerated expansion recently? why are we living in an epoch in which the dark energy density and the dust matter energy density are comparable?* On the other hand, cosmological observations hint that the equation-of-state parameter (EoS) of dark energy crossed the phantom divide  $w_{de} = -1$  in the near past. Many dark energy models whose EoS can cross the phantom divide have been proposed. However, to our knowledge, these models with crossing the phantom divide only provide the possibility that  $w_{de}$  can cross  $-1$ . They do not answer another question, namely, *why crossing the phantom divide occurs recently?* Since in many existing models whose EoS can cross the phantom divide,  $w_{de}$  undulates around  $-1$  randomly, *why are we living in an epoch  $w_{de} < -1$ ?* This can be regarded as the second cosmological coincidence problem. In this work, the cosmological evolution of the vector-like dark energy interacting with background perfect fluid is investigated. We find that the first and second cosmological coincidence problems can be alleviated at the same time in this scenario.

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## I. INTRODUCTION

Dark energy problem [1] has been one of the most active fields in modern cosmology, since the discovery of accelerated expansion of our universe [2, 3, 4, 5, 6, 61]. In the observational cosmology, the equation-of-state parameter (EoS) of dark energy  $w_{de} \equiv p_{de}/\rho_{de}$  plays a central role, where  $p_{de}$  and  $\rho_{de}$  are its pressure and energy density, respectively. To accelerate the expansion, the EoS of dark energy must satisfy  $w_{de} < -1/3$ . The simplest candidate of the dark energy is a tiny positive time-independent cosmological constant  $\Lambda$ , whose EoS is  $-1$ . However, it is difficult to understand why the cosmological constant is about 120 orders of magnitude smaller than its natural expectation, i.e. the Planck energy density. This is the so-called cosmological constant problem. Another puzzle of the dark energy is the (first) cosmological coincidence problem, namely, *why does our universe begin the accelerated expansion recently? why are we living in an epoch in which the dark energy density and the dust matter energy density are comparable?* This problem becomes very serious especially for the cosmological constant as the dark energy candidate. The cosmological constant remains unchanged while the energy densities of dust matter and radiation decrease rapidly with the expansion of our universe. Thus, it is necessary to make some fine-tunings. In order to give a reasonable interpretation to the (first) cosmological coincidence problem, many dynamical dark energy models have been proposed as alternatives to the cosmological constant, such as quintessence [7, 8], phantom [9, 10, 11], k-essence [12] etc.

Recently, by fitting the SNe Ia data, marginal evidence for  $w_{de}(z) < -1$  at redshift  $z < 0.2$  has been found [13]. In addition, many best-fits of the present value of  $w_{de}$  are less than  $-1$  in various data fittings with different parameterizations (see [14] for a recent review). The present observational data seem to slightly favor an evolving dark energy with  $w_{de}$  crossing  $-1$  from above to below in the near past [15]. Obviously, the EoS of dark energy  $w_{de}$  cannot cross the so-called phantom divide  $w_{de} = -1$  for quintessence or phantom alone. Although it seems possible for some variants of k-essence to give a promising solution to cross the phantom divide, a no-go theorem, shown in [16], shatters this kind of hopes. In fact, it is not a trivial task to build dark energy model whose EoS can cross the phantom divide. To this end, a lot of efforts [17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 59] have been made. However, to our knowledge, many of these models only provide the possibility that  $w_{de}$  can cross  $-1$ . They do not answer another question, namely, *why crossing phantom divide occurs recently?* Since in many existing models whose EoS can cross the phantom divide,  $w_{de}$  undulates around  $-1$  randomly, *why are we living in an epoch  $w_{de} < -1$ ?* It can be regarded as the second cosmological coincidence problem [33].

The so-called second cosmological coincidence problem was seriously discussed in [33] for the first time. In [33], the key point is the trigger mechanism, similar to the case of hybrid inflation [34] and the model by Gong and Kim [35]. In the hybrid dark energy model considered in [33], a quintessence and a phantom are employed. The feature of spontaneous symmetry breaking plays a critical role. In the first stage, the phantom field  $\sigma$  is trapped to  $\sigma = 0$ , while the quintessence field  $\phi$  slowly rolls down for a long time. In this stage, the effective EoS of hybrid dark energy remains larger than  $-1$ . When  $\phi$  reaches a critical value  $\phi_c$ , a phase transition takes place and the phantom field  $\sigma$  is triggered to climb up its potential. And then, the quintessence field  $\phi$  is trapped to  $\phi = 0$ . Crossing the phantom divide occurs between the moment of phase transition  $\phi = \phi_c$  and the moment of  $\phi$  being trapped at  $\phi = 0$  eventually. The effective EoS remains smaller than  $-1$  in the major part of the second stage. When  $\sigma$  continuously climbs up, it will eventually arrive at a critical value  $\sigma_c$ . The quintessence field  $\phi$  is triggered to roll down again. In the third stage, whether the EoS remains smaller than  $-1$  or changes to be larger than  $-1$  depends on the profile of the effective potential of hybrid dark energy in this stage. Thus, the avoidance of the big rip is possible for suitable model parameters. We refer to the original paper [33] for more details. (Also, see [60] for other discussion on the second cosmological coincidence problem, soon after our previous paper [33].)

Although the scalar field is used extensively because of its simplicity, the vector field also gets its applications in modern cosmology. Here, we only mention a few, such as in [36, 37], the vector field is considered as the source which drives the inflation; in [38, 39] the Lorentz-violated vector field and its effects to universe are studied; the quantum fluctuations of vector fields produced at the first stage of reheating after inflation is also studied in [40]; the cosmology of massive vector fields with  $SO(3)$  global symmetry is investigated in [41]; and also see [42, 43] and references therein for the literature on the magnetic fields in the universe. It is found that non-linear electromagnetic field can drive the acceleration of the universe [44]. The vector field can be a good dark matter candidate reproducing flat rotation curves

in dark halos of spiral galaxies (see [45] for example). Of course, the vector field is also a viable dark energy candidate [46, 47]. The effects of vector field dark energy candidate on the cosmic microwave background radiation and the large scale structure are discussed in [58].

In this work, the cosmological evolution of the vector-like dark energy proposed in [46] interacting with background perfect fluid is investigated. In fact, in [46] the cosmological evolution of the vector-like dark energy with inverse power-law potential and *without* interaction with background matter was studied, by means of directly numerical and approximate solutions of the equation of motion in two limits of matter domination and vector-like dark energy domination. Differing from [46], we investigate the cosmological evolution of the vector-like dark energy and background perfect fluid in this work by means of dynamical system [48]. And we study the models with not only inverse power-law potential but also exponential potential. Furthermore, we consider the cases of vector-like dark energy interacting with background perfect fluid, while the interaction terms are taken to be four different forms which are familiar in the literature. We find that the first and second cosmological coincidence problems can be alleviated at the same time in this scenario.

This paper is organized as follows. In Sec. II, we will briefly present the main points of the vector-like dark energy model proposed in [46]. In Sec. III, we give out the equations of the dynamical system of vector-like dark energy with interaction to background perfect fluid for the most general case. That is, we leave the potential of vector-like dark energy and the interaction form undetermined. We will investigate the dynamical system for the models with inverse power-law and exponential potentials in Sec. IV and V, respectively. In each case with different potential, we consider four different interaction forms between vector-like dark energy and background perfect fluid. The interaction forms are taken to be the most familiar interaction ones considered in the literature. In Sec. VI, the first and second cosmological coincidence problems are discussed. Finally, brief conclusion and discussion are given in Sec. VII.

We use the units  $\hbar = c = 1$  and  $\kappa^2 \equiv 8\pi G$  throughout this paper.

## II. VECTOR-LIKE DARK ENERGY

In fact, the “vector-like dark energy” is a so-called “cosmic triad” (see [46]), which is a set of three identical vectors pointing in mutually orthogonal directions, in order to avoid violations of isotropy. Following [46], we consider the case of vector-like dark energy minimally coupled to gravity, and the action is given by

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa^2} - \sum_{a=1}^3 \left[ \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + V(A^{a2}) \right] + \mathcal{L}_m(g_{\mu\nu}, \psi) \right\}, \quad (1)$$

where  $g$  is the determinant of the metric  $g_{\mu\nu}$ ,  $R$  is the Ricci scalar,  $F_{\mu\nu}^a \equiv \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$ ,  $A^{a2} \equiv g^{\mu\nu} A_\mu^a A_\nu^a$ , and  $\mathcal{L}_m$  is the Lagrangian density of matter fields  $\psi$ . Latin indices label the different fields ( $a, b, \dots = 1, 2, 3$ ) and Greek indices label different spacetime components ( $\mu, \nu, \dots = 0, 1, 2, 3$ ). Actually, the number of vector fields is dictated by the number of spatial dimensions and the requirement of isotropy [46]. The Latin indices are raised and lowered with the flat “metric”  $\delta_{ab}$ . The potential term  $V(A^{a2})$  explicitly violates gauge invariance. In [46], it is argued that this kind of “cosmic triad” can naturally arise from gauge theory with a single  $SU(2)$  gauge group.

From action (1), one can get the energy-momentum tensor of the cosmic triad and the equations of motion for the vectors  $A_\mu^a$  as

$${}^{(A)}T_{\mu\nu} = \sum_a \left\{ F_{\mu\rho}^a F_\nu^{a\rho} + 2 \frac{dV}{dA^{a2}} A_\mu^a A_\nu^a - \left[ \frac{1}{4} F_{\rho\sigma}^a F^{a\rho\sigma} + V(A^{a2}) \right] g_{\mu\nu} \right\}, \quad (2)$$

and

$$\partial_\mu (\sqrt{-g} F^{a\mu\nu}) = 2\sqrt{-g} \frac{dV}{dA^{a2}} A^{a\nu}, \quad (3)$$

respectively. We consider a spatially flat Friedmann-Robertson-Walker (FRW) universe with metric

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2, \quad (4)$$

where  $a$  is the scale factor. An ansatz for the vectors, which can be compatible with homogeneity and isotropy, is

$$A^b_\mu = \delta^b_\mu A(t) \cdot a. \quad (5)$$

Thus, the three vectors point in mutually orthogonal spatial directions, and share the same time-dependent length, i.e.  $A^2 \equiv A_\mu^a A^{a\mu} = A^2(t)$ . Substituting Eqs. (5) and (4) into Eq. (3), one obtains

$$\ddot{A} + 3H\dot{A} + \left(H^2 + \frac{\ddot{a}}{a}\right)A + \frac{dV}{dA} = 0, \quad (6)$$

where  $H \equiv \dot{a}/a$  is the Hubble parameter, and a dot denotes the derivative with respect to the cosmic time  $t$ . The Friedmann equation and Raychaudhuri equation are given by, respectively,

$$H^2 = \frac{\kappa^2}{3}\rho_{tot} = \frac{\kappa^2}{3}(\rho_A + \rho_m), \quad (7)$$

and

$$\dot{H} = -\frac{\kappa^2}{2}(\rho_{tot} + p_{tot}) = -\frac{\kappa^2}{2}(\rho_A + \rho_m + p_A + p_m), \quad (8)$$

where  $p_m$  and  $\rho_m$  are the pressure and energy density of background matter, respectively. The energy density and (isotropic) pressure of the vector-like dark energy are given by

$$\rho_A = \frac{3}{2}(\dot{A} + HA)^2 + 3V(A^2), \quad (9)$$

$$p_A = \frac{1}{2}(\dot{A} + HA)^2 - 3V(A^2) + 2\frac{dV}{dA^2}A^2, \quad (10)$$

respectively. Noting that  $\ddot{a}/a = H^2 + \dot{H}$ , one can check that Eq. (6) is equivalent to the energy conservation equation of vector-like dark energy, namely  $\dot{\rho}_A + 3H(\rho_A + p_A) = 0$ .

The most remarkable feature of the vector-like dark energy is that its EoS  $w_A \equiv p_A/\rho_A$  can be smaller than  $-1$ , while possessing a conventional positive kinetic term. This is thanks to the additional term in proportion to  $dV/dA^2$  in  $p_A$ . While the energy density  $\rho_A$  is positive, one can find that the condition for  $w_A < -1$  is

$$\frac{dV}{dA^2}A^2 < -(\dot{A} + HA)^2. \quad (11)$$

Thus,  $dV/dA^2 < 0$  is necessary [46, 49]. There are other interesting issues concerning the vector-like dark energy. We refer to the original paper [46] for more details.

### III. DYNAMICAL SYSTEM OF VECTOR-LIKE DARK ENERGY INTERACTING WITH BACKGROUND PERFECT FLUID

As mentioned in Sec. I, in this work we will generalize the original vector-like dark energy model [46] to include the interaction between the vector-like dark energy and background matter. The background matter is described by a perfect fluid with barotropic equation of state

$$p_m = w_m \rho_m \equiv (\gamma - 1)\rho_m, \quad (12)$$

where the barotropic index  $\gamma$  is a constant and satisfies  $0 < \gamma \leq 2$ . In particular,  $\gamma = 1$  and  $4/3$  correspond to dust matter and radiation, respectively. We assume the vector-like dark energy and background matter interact through an interaction term  $C$ , according to

$$\dot{\rho}_A + 3H(\rho_A + p_A) = -C, \quad (13)$$

$$\dot{\rho}_m + 3H(\rho_m + p_m) = C, \quad (14)$$

which preserves the total energy conservation equation  $\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0$ . It is worth noting that the equation of motion (6) should be changed when  $C \neq 0$ , a new term due to  $C$  will appear in its right hand side.

Following [50, 51, 52], we introduce following dimensionless variables

$$x \equiv \frac{\kappa \dot{A}}{\sqrt{6}H}, \quad y \equiv \frac{\kappa \sqrt{V}}{\sqrt{3}H}, \quad z \equiv \frac{\kappa \sqrt{\rho_m}}{\sqrt{3}H}, \quad u \equiv \frac{\kappa A}{\sqrt{6}}. \quad (15)$$

By the help of Eqs. (7)–(10), the evolution equations (13) and (14) can then be rewritten as a dynamical system [48], i.e.

$$x' = 6 \left[ (x+u)^2 + \frac{\gamma}{4}z^2 + \Theta \right] (x+u) - 2\Theta u^{-1} - 3x - 2u - C_1, \quad (16)$$

$$y' = 6y \left[ (x+u)^2 + \frac{\gamma}{4}z^2 + \left( 1 + \frac{1}{3}xy^{-2}u^{-1} \right) \Theta \right], \quad (17)$$

$$z' = 6z \left[ (x+u)^2 + \frac{\gamma}{4}z^2 + \Theta - \frac{\gamma}{4} \right] + C_2, \quad (18)$$

$$u' = x, \quad (19)$$

where

$$\Theta \equiv \frac{u^2}{H^2} \frac{dV}{dA^2}, \quad (20)$$

and

$$C_1 \equiv \frac{\kappa^2 C}{18H^3} (x+u)^{-1}, \quad C_2 \equiv \frac{zC}{2H\rho_m}, \quad (21)$$

a prime denotes derivative with respect to the so-called  $e$ -folding time  $N \equiv \ln a$ , and we have used

$$-\frac{\dot{H}}{H^2} = 6 \left[ (x+u)^2 + \frac{\gamma}{4}z^2 + \Theta \right]. \quad (22)$$

The Friedmann constraint Eq. (7) becomes

$$3 \left[ (x+u)^2 + y^2 \right] + z^2 = 1. \quad (23)$$

The fractional energy densities of vector-like dark energy and background matter are given by

$$\Omega_A \equiv \frac{\kappa^2 \rho_A}{3H^2} = 3 \left[ (x+u)^2 + y^2 \right], \quad \Omega_m \equiv \frac{\kappa^2 \rho_m}{3H^2} = z^2, \quad (24)$$

respectively. The EoS of vector-like dark energy and the effective EoS of the whole system are

$$w_A \equiv \frac{p_A}{\rho_A} = \frac{(x+u)^2 - 3y^2 + 4\Theta}{3 \left[ (x+u)^2 + y^2 \right]}, \quad (25)$$

and

$$w_{eff} \equiv \frac{p_{tot}}{\rho_{tot}} = \Omega_A w_A + \Omega_m w_m, \quad (26)$$

respectively. From Eq. (25), it is easy to see that the condition for  $w_A < -1$  is

$$(x + u)^2 + \Theta < 0, \quad (27)$$

which is equivalent to Eq. (11). Finally, it is worth noting that  $y \geq 0$  and  $z \geq 0$  by definition, and in what follows, we only consider the case of expanding universe with  $H > 0$ .

It is easy to see that Eqs. (16)–(19) become an autonomous system when the potential  $V(A^2)$  is chosen to be an inverse power-law or exponential potential and the interaction term  $C$  is chosen to be a suitable form. Indeed, we will consider the model with an inverse power-law and exponential potential in Sec. IV and V, respectively. In each model with different potential, we consider four cases with different interaction forms between vector-like dark energy and background perfect fluid, which are taken as the most familiar interaction terms extensively considered in the literature [51, 52, 53, 54, 55, 56], namely

$$\text{Case (I)} \quad C = 3\alpha H \rho_m,$$

$$\text{Case (II)} \quad C = 3\beta H \rho_{tot} = 3\beta H (\rho_A + \rho_m),$$

$$\text{Case (III)} \quad C = \eta \kappa \rho_m \dot{A},$$

$$\text{Case (IV)} \quad C = 3\sigma H \rho_A,$$

where  $\alpha, \beta, \eta$  and  $\sigma$  are dimensionless constants.

In the next two sections, we firstly obtain the critical points  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  of the autonomous system by imposing the conditions  $\bar{x}' = \bar{y}' = \bar{z}' = \bar{u}' = 0$ . Of course, they are subject to the Friedmann constraint, namely  $3[(\bar{x} + \bar{u})^2 + \bar{y}^2] + \bar{z}^2 = 1$ . We then discuss the existence and stability of these critical points. An attractor is one of the stable critical points of the autonomous system.

#### IV. MODEL WITH INVERSE POWER-LAW POTENTIAL

In this section, we consider the vector-like dark energy model with an inverse power-law potential

$$V(A^2) = V_0 (\kappa^2 A^2)^{-n}, \quad (28)$$

where  $n$  is a positive dimensionless constant (required by the condition  $dV/dA^2 < 0$ ). In this case,

$$\Theta = -\frac{n}{2}y^2. \quad (29)$$

One can obtain the critical points  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  of the dynamical system Eqs. (16)–(19) with Eqs. (28) and (29) by imposing the conditions  $\bar{x}' = \bar{y}' = \bar{z}' = \bar{u}' = 0$ . Note that these critical points must satisfy the Friedmann constraint (23),  $\bar{y} \geq 0$ ,  $\bar{z} \geq 0$  and the requirement of  $\bar{x}, \bar{y}, \bar{z}, \bar{u}$  all being real. To study the stability of these critical points, we substitute linear perturbations  $x \rightarrow \bar{x} + \delta x$ ,  $y \rightarrow \bar{y} + \delta y$ ,  $z \rightarrow \bar{z} + \delta z$ , and  $u \rightarrow \bar{u} + \delta u$  about the critical point  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  into dynamical system Eqs. (16)–(19) with Eqs. (28) and (29) and linearize them. Because of the Friedmann constraint (23), there are only three independent evolution equations, i.e.

$$\begin{aligned} \delta x' = & \left[ 9(n+2)(\bar{x} + \bar{u})^2 - 2n\bar{u}^{-1}(\bar{x} + \bar{u}) + \left( \frac{3}{2}\gamma + n \right) \bar{z}^2 - n - 3 \right] \delta x \\ & + 2\bar{z} \left[ \left( \frac{3}{2}\gamma + n \right) (\bar{x} + \bar{u}) - \frac{n}{3}\bar{u}^{-1} \right] \delta z + \left[ (18 + 9n + n\bar{u}^{-2}) (\bar{x} + \bar{u})^2 + \left( \frac{3}{2}\gamma + n \right) \bar{z}^2 \right. \\ & \left. - \frac{n}{3} (1 - \bar{z}^2) \bar{u}^{-2} - 2n\bar{x}\bar{u}^{-1} - 3n - 2 \right] \delta u - \delta C_1, \end{aligned} \quad (30)$$

$$\begin{aligned} \delta z' = & 6(n+2)\bar{z}(\bar{x} + \bar{u})\delta x + \left[ 3(n+2)(\bar{x} + \bar{u})^2 + \left( \frac{3}{2}\gamma + n \right) (3\bar{z}^2 - 1) \right] \delta z \\ & + 6(n+2)\bar{z}(\bar{x} + \bar{u})\delta u + \delta C_2, \end{aligned} \quad (31)$$

$$\delta u' = \delta x, \quad (32)$$

where  $\delta C_1$  and  $\delta C_2$  are the linear perturbations coming from  $C_1$  and  $C_2$ , respectively. The three eigenvalues of the coefficient matrix of the above equations determine the stability of the corresponding critical point.

**A. Case (I)  $C = 3\alpha H \rho_m$**

In this case,  $C_1 = \frac{\alpha}{2}(x+u)^{-1}z^2$  and  $C_2 = \frac{3}{2}\alpha z$ . The physically reasonable critical points of the dynamical system Eqs. (16)–(19) with Eqs. (28) and (29) are summarized in Table I. Next, we consider the stability of these critical points. Substituting  $\delta C_1 = -\frac{\alpha}{2}\bar{z}^2(\bar{x}+\bar{u})^{-2}\delta x + \alpha\bar{z}(\bar{x}+\bar{u})^{-1}\delta z - \frac{\alpha}{2}\bar{z}^2(\bar{x}+\bar{u})^{-2}\delta u$ ,  $\delta C_2 = \frac{3}{2}\alpha\delta z$ , and the corresponding critical point  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  into Eqs. (30)–(32), we find that Point (P.I.1) is always unstable; Point (P.I.2) exists and is stable under condition  $\alpha < \gamma$ ; Point (P.I.3) is unstable if it can exist. The unique late time attractor (P.I.2) has

$$\Omega_A = 1, \quad \Omega_m = 0, \quad w_A = -1, \quad w_{eff} = -1, \quad (33)$$

which is a vector-like dark energy dominated solution.

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$
P.I.1	$0, 0, 0, \pm \frac{1}{\sqrt{3}}$
P.I.2	$0, \sqrt{\frac{2}{3(2+n)}}, 0, \pm \sqrt{\frac{n}{3(2+n)}}$
P.I.3	$0, 0, \sqrt{1 - \frac{3\alpha}{3\gamma-4}}, \pm \sqrt{\frac{\alpha}{3\gamma-4}}$

TABLE I: Critical points for Case (I)  $C = 3\alpha H \rho_m$  in the model with inverse power-law potential.

**B. Case (II)  $C = 3\beta H \rho_{tot} = 3\beta H (\rho_A + \rho_m)$**

In this case,  $C_1 = \frac{\beta}{2}(x+u)^{-1}$  and  $C_2 = \frac{3}{2}\beta z^{-1}$ . We present the physically reasonable critical points of the dynamical system Eqs. (16)–(19) with Eqs. (28) and (29) in Table II, where

$$r_1 \equiv \sqrt{1 + \frac{12\beta}{4-3\gamma}}. \quad (34)$$

Then, we substitute  $\delta C_1 = -\frac{\beta}{2}(\bar{x}+\bar{u})^{-2}(\delta x + \delta u)$ ,  $\delta C_2 = -\frac{3}{2}\beta\bar{z}^{-2}\delta z$ , and the corresponding critical point  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  into Eqs. (30)–(32) to study its stability. We find that Point (P.II.1) exists and is stable under conditions  $0 < \beta < \frac{4-3\gamma}{12} \left\{ \left[ \frac{4+3\gamma}{3(3\gamma-4)} \right]^2 - 1 \right\}$  and  $\gamma > 8/3$ , which is out of the range  $0 < \gamma \leq 2$ ; Point (P.II.2) exists and is stable under conditions  $\beta < \min \left\{ 0, \frac{4-3\gamma}{12} \left[ \frac{(4+3\gamma)^2}{9(4-3\gamma)^2} - 1 \right] \right\}$  and  $\gamma < 4/3$ ; Point (P.II.3) exists and is stable in a proper parameter-space [57].

The late time attractor (P.II.2) has

$$\Omega_A = \frac{1}{2}(1 - r_1), \quad \Omega_m = \frac{1}{2}(1 + r_1), \quad w_A = \frac{1}{3}, \quad w_{eff} = \frac{1}{6}[(1 + r_1)(3\gamma - 4) + 2], \quad (35)$$

which is a scaling solution. The late time attractor (P.II.3) has

$$\Omega_A = 1 - \frac{\beta}{\gamma}, \quad \Omega_m = \frac{\beta}{\gamma}, \quad w_A = -1 - \frac{\beta\gamma}{\gamma - \beta}, \quad w_{eff} = -1, \quad (36)$$

which is a scaling solution also. Note that  $w_{eff}$  of attractor (P.II.2) is larger than  $-1$ , since  $0 < \gamma \leq 2$  and  $0 < r_1 < 1$  which is required by Eq. (34) and its corresponding  $\Omega_m$ , while  $w_A$  of attractor (P.II.3) is smaller than  $-1$ , since  $0 < \beta < \gamma$  is required by its corresponding  $\Omega_m$ .

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$
P.II.1	$0, 0, [\frac{1}{2}(1-r_1)]^{1/2}, \pm [\frac{1}{6}(1+r_1)]^{1/2}$
P.II.2	$0, 0, [\frac{1}{2}(1+r_1)]^{1/2}, \pm [\frac{1}{6}(1-r_1)]^{1/2}$
P.II.3	$0, \sqrt{\frac{4\gamma+\beta(-4+3\gamma)}{6(2+n)\gamma}}, \sqrt{\frac{\beta}{\gamma}}, \pm \sqrt{\frac{2n(\gamma-\beta)-3\beta\gamma}{6(2+n)\gamma}}$

TABLE II: Critical points for Case (II)  $C = 3\beta H \rho_{tot} = 3\beta H (\rho_A + \rho_m)$  in the model with inverse power-law potential.  $r_1$  is given in Eq. (34).

### C. Case (III) $C = \eta \kappa \rho_m \dot{A}$

In this case,  $C_1 = \frac{\eta}{\sqrt{6}}(x+u)^{-1}xz^2$  and  $C_2 = \sqrt{\frac{3}{2}}\eta xz$ . The physically reasonable critical points of the dynamical system Eqs. (16)–(19) with Eqs. (28) and (29) are shown in Table III. Substituting  $\delta C_1 = \frac{\eta}{\sqrt{6}}(\bar{x}+\bar{u})^{-1}\bar{z}^2 [1 - (\bar{x}+\bar{u})^{-1}\bar{x}] \delta x + \sqrt{\frac{2}{3}}\eta(\bar{x}+\bar{u})^{-1}\bar{x}\bar{z}\delta z - \frac{\eta}{\sqrt{6}}(\bar{x}+\bar{u})^{-2}\bar{x}\bar{z}^2\delta u$ ,  $\delta C_2 = \sqrt{\frac{3}{2}}\eta\bar{z}\delta x + \sqrt{\frac{3}{2}}\eta\bar{x}\delta z$ , and the corresponding critical point  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  into Eqs. (30)–(32), we find that Point (P.III.1) is always unstable, while Point (P.III.2) is always stable. The unique late time attractor (P.III.2) has

$$\Omega_A = 1, \quad \Omega_m = 0, \quad w_A = -1, \quad w_{eff} = -1, \quad (37)$$

which is a vector-like dark energy dominated solution.

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$
P.III.1	$0, 0, 0, \pm \frac{1}{\sqrt{3}}$
P.III.2	$0, \sqrt{\frac{2}{3(2+n)}}, 0, \pm \sqrt{\frac{n}{3(2+n)}}$

TABLE III: Critical points for Case (III)  $C = \eta \kappa \rho_m \dot{A}$  in the model with inverse power-law potential.

### D. Case (IV) $C = 3\sigma H \rho_A$

In this case,  $C_1 = \frac{3}{2}\sigma [(x+u)^2 + y^2] (x+u)^{-1} = \frac{\sigma}{2}(1-z^2)(x+u)^{-1}$  and  $C_2 = \frac{9}{2}\sigma [(x+u)^2 + y^2] z^{-1} = \frac{3}{2}\sigma (z^{-1} - z)$ . The physically reasonable critical points of the dynamical system Eqs. (16)–(19) with Eqs. (28) and (29) are shown in Table IV, where

$$r_2 \equiv \frac{\sigma}{-4+3\gamma}. \quad (38)$$

Next, we consider the stability of these critical points. Substituting  $\delta C_1 = -\frac{\sigma}{2}(\bar{x}+\bar{u})^{-2}(1-\bar{z}^2)\delta x - \sigma\bar{z}(\bar{x}+\bar{u})^{-1}\delta z - \frac{\sigma}{2}(\bar{x}+\bar{u})^{-2}(1-\bar{z}^2)\delta u$ ,  $\delta C_2 = -\frac{3}{2}\sigma(\bar{z}^{-2}+1)\delta z$ , and the corresponding critical point  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  into Eqs. (30)–(32), we find that Point (P.IV.1) is unstable if it can exist, while Point (P.IV.2) exists and is stable in a proper parameter-space [57]. The unique late time attractor (P.IV.2) has

$$\Omega_A = \frac{\gamma}{\gamma+\sigma}, \quad \Omega_m = \frac{\sigma}{\gamma+\sigma}, \quad w_A = -1-\sigma, \quad w_{eff} = -1, \quad (39)$$

which is a scaling solution. Note that  $w_A$  of attractor (P.IV.2) is smaller than  $-1$ , since  $\sigma > 0$  is required by its corresponding  $\Omega_m$ .



Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$
P.IV.1	$0, 0, (3r_2)^{1/2}, \pm (\frac{1}{3} - r_2)^{1/2}$
P.IV.2	$0, \sqrt{\frac{\gamma(4+3\sigma)}{6(2+n)(\gamma+\sigma)}}, \sqrt{\frac{\sigma}{\gamma+\sigma}}, \pm \sqrt{\frac{\gamma(2n-3\sigma)}{6(2+n)(\gamma+\sigma)}}$

TABLE IV: Critical points for Case (IV)  $C = 3\sigma H\rho_A$  in the model with inverse power-law potential.  $r_2$  is given in Eq. (38).

## V. MODEL WITH EXPONENTIAL POTENTIAL

In this section, we consider the vector-like dark energy model with an exponential potential

$$V(A^2) = V_0 \exp(-\lambda \kappa^2 A^2), \quad (40)$$

where  $\lambda$  is a positive dimensionless constant (required by the condition  $dV/dA^2 < 0$ ). In this case,

$$\Theta = -3\lambda u^2 y^2. \quad (41)$$

One can obtain the critical points  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  of the dynamical system Eqs. (16)–(19) with Eqs. (40) and (41) by imposing the conditions  $\bar{x}' = \bar{y}' = \bar{z}' = \bar{u}' = 0$ . Note that these critical points must satisfy the Friedmann constraint (23),  $\bar{y} \geq 0$ ,  $\bar{z} \geq 0$  and the requirement of  $\bar{x}, \bar{y}, \bar{z}, \bar{u}$  all being real. To study the stability of these critical points, we substitute linear perturbations  $x \rightarrow \bar{x} + \delta x$ ,  $y \rightarrow \bar{y} + \delta y$ ,  $z \rightarrow \bar{z} + \delta z$ , and  $u \rightarrow \bar{u} + \delta u$  about the critical point  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  into dynamical system Eqs. (16)–(19) with Eqs. (40) and (41) and linearize them. Because of the Friedmann constraint (23), there are only three independent evolution equations, namely

$$\begin{aligned} \delta x' = & 6 \left[ 3(1 + 3\lambda \bar{u}^2)(\bar{x} + \bar{u})^2 - 2\lambda \bar{u}(\bar{x} + \bar{u}) + \left( \lambda \bar{u}^2 + \frac{\gamma}{4} \right) \bar{z}^2 - \lambda \bar{u}^2 - \frac{1}{2} \right] \delta x \\ & + 4\bar{z} \left[ 3(\bar{x} + \bar{u}) \left( \lambda \bar{u}^2 + \frac{\gamma}{4} \right) - \lambda \bar{u} \right] \delta z + 2 \left\{ 18\lambda \bar{u}(\bar{x} + \bar{u})^3 + 3(9\lambda \bar{u}^2 + 3 - \lambda)(\bar{x} + \bar{u})^2 \right. \\ & \left. - 6\lambda \bar{u}(\bar{x} + \bar{u}) + \left[ 6\lambda \bar{u}(\bar{x} + \bar{u}) + 3 \left( \lambda \bar{u}^2 + \frac{\gamma}{4} \right) - \lambda \right] (\bar{z}^2 - 1) + \frac{3}{4}\gamma - 1 \right\} \delta u - \delta C_1, \end{aligned} \quad (42)$$

$$\begin{aligned} \delta z' = & 12\bar{z} (1 + 3\lambda \bar{u}^2) (\bar{x} + \bar{u}) \delta x + 6 \left[ (1 + 3\lambda \bar{u}^2) (\bar{x} + \bar{u})^2 + \left( \lambda \bar{u}^2 + \frac{\gamma}{4} \right) (3\bar{z}^2 - 1) \right] \delta z \\ & + 12\bar{z} [3\lambda \bar{u}(\bar{x} + \bar{u})^2 + (1 + 3\lambda \bar{u}^2)(\bar{x} + \bar{u}) + \lambda \bar{u}(\bar{z}^2 - 1)] \delta u + \delta C_2, \end{aligned} \quad (43)$$

$$\delta u' = \delta x, \quad (44)$$

where  $\delta C_1$  and  $\delta C_2$  are the linear perturbations coming from  $C_1$  and  $C_2$ , respectively. The three eigenvalues of the coefficient matrix of the above equations determine the stability of the corresponding critical point.

### A. Case (I) $C = 3\alpha H\rho_m$

In this case,  $C_1 = \frac{\alpha}{2}(x+u)^{-1}z^2$  and  $C_2 = \frac{3}{2}\alpha z$ . The physically reasonable critical points of the dynamical system Eqs. (16)–(19) with Eqs. (40) and (41) are summarized in Table V. Next, we consider the stability of these critical points. Substituting  $\delta C_1 = -\frac{\alpha}{2}\bar{z}^2(\bar{x}+\bar{u})^{-2}\delta x + \alpha\bar{z}(\bar{x}+\bar{u})^{-1}\delta z - \frac{\alpha}{2}\bar{z}^2(\bar{x}+\bar{u})^{-2}\delta u$ ,  $\delta C_2 = \frac{3}{2}\alpha\delta z$ , and the corresponding critical point  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  into Eqs. (42)–(44), we find that Point (E.I.1) is always unstable; Point (E.I.2) exists and is stable under conditions  $\alpha < \gamma$  and  $\lambda > 1$ ; Point (E.I.3) is unstable if it can exist. The unique late time attractor (E.I.2) has

$$\Omega_A = 1, \quad \Omega_m = 0, \quad w_A = -1, \quad w_{eff} = -1, \quad (45)$$

which is a vector-like dark energy dominated solution.

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$
E.I.1	$0, 0, 0, \pm \frac{1}{\sqrt{3}}$
E.I.2	$0, \frac{1}{\sqrt{3\lambda}}, 0, \pm \sqrt{\frac{\lambda-1}{3\lambda}}$
E.I.3	$0, 0, \sqrt{1 - \frac{3\alpha}{3\gamma-4}}, \pm \sqrt{\frac{\alpha}{3\gamma-4}}$

TABLE V: Critical points for Case (I)  $C = 3\alpha H \rho_m$  in the model with exponential potential.

**B. Case (II)  $C = 3\beta H \rho_{tot} = 3\beta H (\rho_A + \rho_m)$**

In this case,  $C_1 = \frac{\beta}{2}(x+u)^{-1}$  and  $C_2 = \frac{3}{2}\beta z^{-1}$ . We present the physically reasonable critical points of the dynamical system Eqs. (16)–(19) with Eqs. (40) and (41) in Table VI, where

$$r_3 \equiv \sqrt{-3\beta\gamma^2\lambda + (\gamma + \beta\lambda - \gamma\lambda)^2}. \quad (46)$$

Then, we substitute  $\delta C_1 = -\frac{\beta}{2}(\bar{x} + \bar{u})^{-2}(\delta x + \delta u)$ ,  $\delta C_2 = -\frac{3}{2}\beta\bar{z}^{-2}\delta z$ , and the corresponding critical point  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  into Eqs. (42)–(44) to study its stability. We find that Point (E.II.1) exists and is stable under conditions  $0 < \beta < \frac{4-3\gamma}{12} \left\{ \left[ \frac{4+3\gamma}{3(3\gamma-4)} \right]^2 - 1 \right\}$  and  $\gamma > 8/3$ , which is out of the range  $0 < \gamma \leq 2$ ; Point (E.II.2) exists and is stable under conditions  $\beta < \min \left\{ 0, \frac{4-3\gamma}{12} \left[ \frac{(4+3\gamma)^2}{9(4-3\gamma)^2} - 1 \right] \right\}$  and  $\gamma < 4/3$ ; Points (E.II.3) and (E.II.4) exist and are stable in a proper parameter-space [57], respectively.

The late time attractor (E.II.2) has

$$\Omega_A = \frac{1}{2}(1 - r_1), \quad \Omega_m = \frac{1}{2}(1 + r_1), \quad w_A = \frac{1}{3}, \quad w_{eff} = \frac{1}{6}[(1 + r_1)(3\gamma - 4) + 2], \quad (47)$$

which is a scaling solution. The late time attractors (E.II.3) and (E.II.4) both have

$$\Omega_A = 1 - \frac{\beta}{\gamma}, \quad \Omega_m = \frac{\beta}{\gamma}, \quad w_A = -1 - \frac{\beta\gamma}{\gamma - \beta}, \quad w_{eff} = -1, \quad (48)$$

which are scaling solutions also. Note that  $w_{eff}$  of attractor (E.II.2) is larger than  $-1$ , since  $0 < \gamma \leq 2$  and  $0 < r_1 < 1$  which is required by Eq. (34) and its corresponding  $\Omega_m$ , while  $w_A$  of attractors (E.II.3) and (E.II.4) are both smaller than  $-1$ , since  $0 < \beta < \gamma$  is required by their corresponding  $\Omega_m$ .

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$
E.II.1	$0, 0, \left[\frac{1}{2}(1 - r_1)\right]^{1/2}, \pm \left[\frac{1}{6}(1 + r_1)\right]^{1/2}$
E.II.2	$0, 0, \left[\frac{1}{2}(1 + r_1)\right]^{1/2}, \pm \left[\frac{1}{6}(1 - r_1)\right]^{1/2}$
E.II.3	$0, \sqrt{\frac{\gamma - \beta\lambda + \gamma\lambda - r_3}{6\gamma\lambda}}, \sqrt{\frac{\beta}{\gamma}}, \pm \sqrt{\frac{\gamma(-1+\lambda) - \beta\lambda + r_3}{6\gamma\lambda}}$
E.II.4	$0, \sqrt{\frac{\gamma - \beta\lambda + \gamma\lambda + r_3}{6\gamma\lambda}}, \sqrt{\frac{\beta}{\gamma}}, \pm \sqrt{\frac{\gamma(-1+\lambda) - \beta\lambda - r_3}{6\gamma\lambda}}$

TABLE VI: Critical points for Case (II)  $C = 3\beta H \rho_{tot} = 3\beta H (\rho_A + \rho_m)$  in the model with exponential potential.

$r_1$  and  $r_3$  are given in Eqs. (34) and (46), respectively.

### C. Case (III) $C = \eta\kappa\rho_m\dot{A}$

In this case,  $C_1 = \frac{\eta}{\sqrt{6}}(x+u)^{-1}xz^2$  and  $C_2 = \sqrt{\frac{3}{2}}\eta xz$ . The physically reasonable critical points of the dynamical system Eqs. (16)–(19) with Eqs. (40) and (41) are shown in Table VII. Substituting  $\delta C_1 = \frac{\eta}{\sqrt{6}}(\bar{x}+\bar{u})^{-1}\bar{z}^2 [1 - (\bar{x}+\bar{u})^{-1}\bar{x}] \delta x + \sqrt{\frac{2}{3}}\eta(\bar{x}+\bar{u})^{-1}\bar{x}\bar{z}\delta z - \frac{\eta}{\sqrt{6}}(\bar{x}+\bar{u})^{-2}\bar{x}\bar{z}^2\delta u$ ,  $\delta C_2 = \sqrt{\frac{3}{2}}\eta\bar{z}\delta x + \sqrt{\frac{3}{2}}\eta\bar{x}\delta z$ , and the corresponding critical point  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  into Eqs. (42)–(44), we find that Point (E.III.1) is always unstable, while Point (E.III.2) exists and is stable under condition  $\lambda > 1$ . The unique late time attractor (E.III.2) has

$$\Omega_A = 1, \quad \Omega_m = 0, \quad w_A = -1, \quad w_{eff} = -1, \quad (49)$$

which is a vector-like dark energy dominated solution.

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$
E.III.1	$0, 0, 0, \pm \frac{1}{\sqrt{3}}$
E.III.2	$0, \frac{1}{\sqrt{3\lambda}}, 0, \pm \sqrt{\frac{\lambda-1}{3\lambda}}$

TABLE VII: Critical points for Case (III)  $C = \eta\kappa\rho_m\dot{A}$  in the model with exponential potential.

### D. Case (IV) $C = 3\sigma H\rho_A$

In this case,  $C_1 = \frac{3}{2}\sigma [(x+u)^2 + y^2] (x+u)^{-1} = \frac{\sigma}{2}(1-z^2)(x+u)^{-1}$  and  $C_2 = \frac{9}{2}\sigma [(x+u)^2 + y^2] z^{-1} = \frac{3}{2}\sigma (z^{-1} - z)$ . The physically reasonable critical points of the dynamical system Eqs. (16)–(19) with Eqs. (40) and (41) are shown in Table VIII, where

$$r_4 \equiv \sqrt{-3\gamma\lambda\sigma(\gamma+\sigma) + (\gamma - \gamma\lambda + \sigma)^2}. \quad (50)$$

Then, we consider the stability of these critical points. Substituting  $\delta C_1 = -\frac{\sigma}{2}(\bar{x}+\bar{u})^{-2} (1 - \bar{z}^2) \delta x - \sigma\bar{z}(\bar{x}+\bar{u})^{-1}\delta z - \frac{\sigma}{2}(\bar{x}+\bar{u})^{-2} (1 - \bar{z}^2) \delta u$ ,  $\delta C_2 = -\frac{3}{2}\sigma (\bar{z}^{-2} + 1) \delta z$ , and the corresponding critical point  $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$  into Eqs. (42)–(44), we find that Point (E.IV.1) is unstable if it can exist, while Points (E.IV.2) and (E.IV.3) exist and are stable in a proper parameter-space [57], respectively. The late time attractors (E.IV.2) and (E.IV.3) both have

$$\Omega_A = \frac{\gamma}{\gamma+\sigma}, \quad \Omega_m = \frac{\sigma}{\gamma+\sigma}, \quad w_A = -1 - \sigma, \quad w_{eff} = -1, \quad (51)$$

which are scaling solutions. Note that  $w_A$  of attractors (E.IV.2) and (E.IV.3) are both smaller than  $-1$ , since  $\sigma > 0$  is required by their corresponding  $\Omega_m$ .

## VI. THE FIRST AND SECOND COSMOLOGICAL COINCIDENCE PROBLEMS

As is well known, the most frequently used approach to alleviate the (first) cosmological coincidence problem is the scaling attractor(s) in the interacting dark energy scenario (see [48, 50, 51, 52, 53, 54, 55, 56] for examples). The dark energy can exchange energy with the background matter (usually the cold dark matter), through the interaction between them. The most desirable feature of dynamical system is that the whole system will eventually evolve to its attractors, having nothing to do with the initial conditions.

Label	Critical Point $(\bar{x}, \bar{y}, \bar{z}, \bar{u})$
E.IV.1	$0, 0, (3r_2)^{1/2}, \pm (\frac{1}{3} - r_2)^{1/2}$
E.IV.2	$0, \sqrt{\frac{\gamma(1+\lambda)+\sigma-r_4}{6\lambda(\gamma+\sigma)}}, \sqrt{\frac{\sigma}{\gamma+\sigma}}, \pm \sqrt{\frac{\gamma(-1+\lambda)-\sigma+r_4}{6\lambda(\gamma+\sigma)}}$
E.IV.3	$0, \sqrt{\frac{\gamma(1+\lambda)+\sigma+r_4}{6\lambda(\gamma+\sigma)}}, \sqrt{\frac{\sigma}{\gamma+\sigma}}, \pm \sqrt{\frac{\gamma(-1+\lambda)-\sigma-r_4}{6\lambda(\gamma+\sigma)}}$

TABLE VIII: Critical points for Case (IV)  $C = 3\sigma H\rho_A$  in the model with exponential potential.  $r_2$  and  $r_4$  are given in Eqs. (38) and (50), respectively.

Therefore, fine-tunings are needless. When the universe is attracted into the scaling attractor, a balance can be achieved, thanks to the interaction. In the scaling attractor, the effective densities of dark energy and background matter decrease in the same manner with the expansion of our universe, and the ratio of dark energy and background matter becomes a constant. So, it is not strange that we are living in an epoch when the densities of dark energy and matter are comparable. In this sense, the (first) cosmological coincidence problem is alleviated (see [48, 50, 51, 52, 53, 54, 55, 56] for examples).

On the other hand, if the scaling attractor also has the property that its EoS of dark energy is smaller than  $-1$ , the second cosmological coincidence problem is alleviated at the same time. However, this is impossible in the interacting quintessence or k-essence scenario. Although the attractor's EoS is smaller than  $-1$  in the interacting scalar phantom scenario, it is impossible to cross the phantom divide  $w_{de} = -1$ , since the EoS of scalar phantom is always smaller than  $-1$ . Fortunately, the EoS of vector-like dark energy can be smaller than  $-1$ , while possessing a conventional positive kinetic term [46], in contrast to the scalar phantom. Of course, the EoS of vector-like dark energy can be larger than  $-1$  also. Thus, crossing the phantom divide is possible in the vector-like dark energy model [46]. As is explicitly shown in this work, for suitable interaction forms [for instance, Cases (II) and (IV)], regardless of the model with inverse power-law or exponential potential, there are some attractors with  $w_A < -1$  while their corresponding  $\Omega_A$  and  $\Omega_m$  are comparable in the interacting vector-like dark energy model. In the Case (IV), *all* stable attractors have these desirable properties. Even in the Case (II), we can choose the model parameters to avoid the attractor with  $w_A > -1$ , and the scaling attractor(s) with  $w_A < -1$  becomes the *unique* late time attractor(s). So, for a fairly wide range of initial conditions with  $w_A > -1$ , the universe will eventually evolve to the scaling attractor(s) with  $w_A < -1$ . Similar to the ordinary way to alleviate the (first) cosmological coincidence problem, the second cosmological coincidence problem is alleviated at the same time.

## VII. CONCLUSION AND DISCUSSION

In summary, the cosmological evolution of the vector-like dark energy interacting with background perfect fluid is investigated in this work. We find that the first and second cosmological coincidence problems can be alleviated at the same time in this scenario. Our results obtained here may support the vector field to be one of the viable dark energy candidates. In particular, the feature of the vector-like dark energy that its EoS  $w_A \equiv p_A/\rho_A$  can be smaller than  $-1$  while possessing a conventional positive kinetic term is very attractive. While considering the interaction between the vector-like dark energy and background matter, the first and second cosmological coincidence problems can be alleviated at the same time. This is a profitable support to the vector-like dark energy. Of course, there are many remaining works to make this scenario more concrete, especially to fit the observational data to determine the realistic model parameters, which is beyond the main aim of the present work.

The other issue is concerning the fate of our universe. It is easy to see that for all cases considered in this work, *all* stable attractors have  $w_{eff} \geq -1$ . Although the EoS of vector-like dark energy can be smaller than  $-1$ , the big rip never appears in this model. This is also in contrast to the ordinary phantom-like models.

Finally, we would like to mention that the gauge invariance is violated for the potential forms  $V(A^{a^2})$  taken in this paper. Although in [46], it is argued that this kind of “cosmic triad” can naturally arise from gauge theory with a single  $SU(2)$  gauge group because of the equations of motion can be written down in a gauge invariant form, one should be careful to this problem. For the cases of inverse power-law and exponential potential, however, it does no longer hold, which means a particular gauge has been taken.

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## APPENDIX

The particular parameter-space for the existence and stability of these critical points is considerably involved and verbose. Since our main aim here is just to point out the fact that it can exist and is stable, we do not present those very long expressions for the corresponding parameter-space. Of course, one can easily work out with the help of Mathematica.

Instead of giving concrete expressions, we here give just some examples to support our statement. For the case of inverse power-law potential (Sec. IV), Point (P.II.3) of Case (II) exists and is stable for parameters  $\gamma = 1$ ,  $\beta = 1/3$ , and  $n = 2$ , with eigenvalues  $\{-0.866239, -1.36688 - i 3.29093, -1.36688 + i 3.29093\}$ , while it is unstable for parameters  $\gamma = 1$ ,  $\beta = 1/3$ , and  $n = 1$ , with eigenvalues  $\{-1.26235, 2.13117 - i 2.04255, 2.13117 + i 2.04255\}$ ; Point (P.IV.2) of Case (IV) exists and is stable for parameters  $\gamma = 1$ ,  $\sigma = 1/2$ , and  $n = 2$ , with eigenvalues  $\{-1.70481, -1.6976 + i 2.60705, -1.6976 - i 2.60705\}$ , and for parameters  $\gamma = 1$ ,  $\sigma = 1/2$ , and  $n = 3$ , with eigenvalues  $\{-1.1769, -2.32821 + i 2.93245, -2.32821 - i 2.93245\}$ , while it is unstable for parameters  $\gamma = 1$ ,  $\sigma = 1/2$ , and  $n = 1$ , with eigenvalues  $\{2.15106 - i 1.12309, -2.80212, 2.15106 + i 1.12309\}$ . Similarly, for the case of exponential potential (Sec. V), Point (E.II.3) of Case (II) exists and is stable for parameters  $\gamma = 1$ ,  $\beta = 1/3$ , and  $\lambda = 6$ , while Point (E.II.4) is unstable for the same parameters; Point (E.IV.2) of Case (IV) exists and is stable for parameters  $\gamma = 1$ ,  $\sigma = 1/2$ , and  $\lambda = 5$ , while Point (E.IV.3) is unstable for the same parameters. Note that in the above examples, we chose the demonstrative parameter  $\gamma = 1$  for dust matter, and  $\beta = 1/3$  or  $\sigma = 1/2$  to make  $\Omega_m = 1/3$ , which are more realistic.

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